**A DIALOGUE ON DIALOGUE**

**Paul Ernest & Anna Sfard**

**From:** Ernest, Paul [mailto:P.Ernest@exeter.ac.uk]   
**Sent:** Thursday, March 30, 2017 3:51 PM  
**To:** Anna Sfard

*Dear Anna*

*Re commognition - can you point me to an account of yours that deals with thinking and the stuff that seems private to us? I ask out of personal curiosity - I too draw on Vygotsky and Wittgenstein - but am currently worrying about imagination!*

Very best wishes

*Paul*

**From:** Anna Sfard [mailto:sfard@netvision.net.il]   
**Sent:** Saturday, April 1, 2017 2:53 PM  
**To:** 'Ernest, Paul'

Hi Paul,

The question, the way you put it, may be interpreted in several ways, so I decided to choose one. You will latter tell me whether this is what you meant and if not, then 'll try to answer it again.

I read you as inquiring about how we go, conceptually and methodologically, around the fact that our thinking is inaccessible to third-party inspection. Is this right? Should we call it an inherently private phenomenon and give up trying to investigate it, the way behaviorists suggested? Or rather should we do what cognitivists suggested we might: recognize the uniqueness of mental phenomena and try to deduce their properties from what is publicly accessible and can be observed?

Here is what I wrote in my book, after Wittgenstein, on these two options:

In the last two decades of his life, Wittgenstein rejected the monological vision of language as a reflection of things in the world, the view for which he was arguing just a few years earlier. In most cases, he now claimed, the meaning of words does not arise from objects signified by these words; rather, “for a *large* class of cases – though not for all – in which we employ the word ‘meaning’ it can be defined thus: the meaning of a word is its use in the language.” Thus, for Wittgenstein, meaning was neither a thing in the world nor a private entity in one’s mind: It was an aspect of human discursive activity and, as such was public and fully investigable.

Consequently, Wittgenstein regarded the behaviorist doctrine as a gross misinterpretation of what the call for disobjectification in general, and the protest against separating thinking from other forms of human doing in particular, were all about. He distanced himself explicitly from behaviorism by engaging in a fictitious dialogue with a critic. When asked by an imaginary interlocutor, “‘Are you not really a behaviourist in disguise? Aren’t you at bottom really saying that everything except human behaviour is a fiction?’” he answered, “If I do speak of a fiction then it is of a *grammatical* fiction.” The idiosyncratically used adjective *grammatical* was meant to indicate, once again, that whatever he said referred to language and its misuses, and not to what does or does not exist in reality or what is or is not empirically investigable. For Wittgenstein, the real dichotomy was in the discourse, not in the world: It was the dichotomy between what is communicationally operative and what is doomed to remain vague and misleading. If we thus need to ban anything, it is not the study of certain phenomena; a ban, if any, should be on things that we mention but that will always result in logical entanglements.

On the other hand, Wittgenstein believed that the things that are open to investigation in a publicly accountable way are much more numerous and incomparably more complex than those included in monological research so far, either behaviorist or cognitivist. Whereas adamant in his claim “What we cannot speak about [properly] we must pass over in silence,” Wittgenstein included in the category of “communicables” more than dreamed of by any monological psychologist. Rather than reducing humans to input–output machines, as behaviorists did, he posited utmost complexity of human actions. Although this claim seemingly placed him closer to cognitivists, his insistence on the inherently public nature of all this complexity made it clear that, in fact, he would not have joined the “cognitive revolution” had he lived long enough to see it. Indeed, rather than insisting that external behavior is but a “tip of an iceberg” whereas the “real action” is elsewhere, as proposed by cognitivists, he sustained that whatever matters in human activities lies open to public inspection. These activities should therefore be taken for what they appear to be: *an extremely complex network of communally established games, which are communicable by definition*.

Assuming stable inner entities responsible for these intricate phenomena would be another attempt at reduction, comparable to that of behaviorists. Rather than looking for a hidden simplifying mechanism, we should deal with this complexity in its own terms. Thus, the observable phenomena that occasion the talk about thinking are not mere “windows” to another, inherently private universe. Rather, these are objects of investigation in their own right, and exclusively so. To put it in Wittgenstein’s own words,

The phenomenon is not a symptom of something else; rather, it is reality. The phenomenon is not a symptom of something else which then makes the sentence true or false; rather, it is itself that which verifies it [the sentence].

This said, one should also remember that the complexity of different forms of human doing is neither modular nor describable with a finite set of rules. None of the human actions can be usefully considered separately from the “forms of life” of which they are inextricable parts and that, although public and investigable, are messy and unyielding and doomed to remain a perennial challenge to those who seek all-encompassing theories of human processes. (Sfard, 2008, pp. 73-74)

So, Wittgenstein proposed to reject both behaviorism and cognitivism, seeing them as basically dualistic: both propose that human phenomena can be divided into physical and mental, and the only difference between them is that one bans the investigation of the mental, whereas the other obliges its investigation, if only indirect.

But if together with Wittgenstein we opt for non-dualist vision of things, what does it tell us about thinking, activity which is inherently personal, and in which all of us engage for no less than all our waking hours? The question is both about *how to conceptualize* *thinking in non-dualist terms* and how to make the definition *operational*, that is, expressed in terms that make it possible for one person to investigate thinking of another. The commognitive answer is inherently non-dualist (monistic): defined as a (potentially multimodal) conversation a person has with herself, thinking falls into the same ontological category as publicly accessible, inherently physical, interpersonal communication. Thus, in investigating thinking, you draw on what you know about human communication. And the fact that much of the thinking process is inaccessible to an observer? Well, we are used to this kind of situation in science: here, we treat some things that we see as constituting a part of a greater whole, and we see this greater whole as composed of a single type of entities – the same type of entities compose the visible and invisible parts. When we interview a person in the hope to investigate her thinking, there are parts of the event – those parts that involve the person's thinking – which are not accessible to us. The main point about this is that while conjecturing about the missing parts, we do not assume that they are ontologically distinct from those to which we do have an access. Or, as I put it in the chapter for your book[[1]](#footnote-1)

researcher who makes conjectures about the learner's inner dialogues may be compared to the archeologists who reconstructs an ancient bowl by complementing the excavated pieces with her own additions. In both cases, the observable and the added pieces are of the same kind, and are informed by the researcher's deep understanding of the relevant context and by her awareness of what would count as reasonable. In this respect, commognitive research differs from cognitivist studies, in which thinking is treated as a different, and more basic kind of phenomenon than interpersonal communication. (Sfard, 2008.

As I made clear from the beginning, I'm not sure I interpreted your question the way I should, but if I did, I hope my answer makes some sense to you. And if I am wrong on any of these two accounts, please ask again and I'll be happy to try to do better.

I am intrigued by your study of imagination. Anything cooked enough to be shared?

Thanks Paul. And till the next time,

Anna

**From:** Ernest, Paul [<mailto:P.Ernest@exeter.ac.uk>]   
**Sent:** Tuesday, April 4, 2017 4:18 PM  
**To:** Anna Sfard

*Hi Anna*

*Thanks for your reply - you didn't quite address my issue. I guess I didn't express it clearly. I accept that internalised dialogue is partially the origin of thought - as Vygotsky says. But from a social, publicly observable account of meaning, reasoning etc - which I accept - how can we account for the phenomenology of thinking. Stuff goes on in my mind without me talking or listening. I know my thinking has been conditioned by dialogue but we experience something more - attention, imagination, visualisation wrapped into and around our internal dialogue. How do we theorize this? Vygotsky acknowledge that our thinking is shaped differently from speech. And of course we don't line up out thoughts and then eject them as speech in order! I know I'm using inner and outer - but I mean private and public. And although we don't have a private language (after Wittgenstein) yet we have private thinking!*

Best wishes

*Paul*

**From:** Anna Sfard <[sfard@netvision.net.il](mailto:sfard@netvision.net.il)>  
**Sent:** 11 April 2017 11:12  
**To:** Ernest, Paul

Hi again Paul,

I have been traveling and this is why have not responded your last email in a timely manner. Well, I’m not sure you intended to continue this conversation, but I find the issues you raise interesting, so won’t keep quiet (and if all this prattle, on my part, is more than you gambled for, you can always ignore).

I think we are here in the midst of a typical philosophical discussion, where, to use Wittgenstein’s language, the two participants seem to be falling into linguistic traps – into communicational pitfalls. At the moment, I can see two problems: (1) I am not sure what questions you are asking (2) I think that you may be worried about the issue of privacy for wrong reasons. Let me explain.

So first, about your question. You ask: “*how can we account for the phenomenology of thinking?”* I am not sure what “accounting for” means in this context, even though you seem to explain:

*“Stuff goes on in my mind without me talking or listening. I know my thinking has been conditioned by dialogue but we experience something more - attention, imagination, visualisation wrapped into and around our internal dialogue. How do we theorize this?”*

Here, “accounting for” turned into “theorizing”: you are talking about *theorizing* our private experiences. This seems to be a request for, first, the possibility to communicate these experiences to others (otherwise, how do you “theorize *this*”? The “this” has to be described in language) and then to explain how they come into being and what role they play in our way of acting in the world. My ex-student, Einat Heyd-Metzuyanim, has been grappling with all these questions while trying to capture the role of emotions in learning. Her research was not just theoretical – she needed the theory to be able to respond to this challenge with an *empirical* investigation (see e.g. Heyd-Metzuyanim, E., & Sfard, A. (2012). Identity struggles in the mathematics classroom: On learning mathematics as an interplay of mathematizing and identifying. *International Journal of Educational Research, 51*, 128-145). From our experience, I do know that the task Einat took upon herself is dauntingly difficult and we are far from satisfied with the theoretical mixture we have managed to concoct, so far. So, I am not surprised by the fact that you sound troubled – we are troubled ourselves. This said, I suspect that you may be worried for a wrong reason! Let me explain.

You say: *“And although we don't have a private language (after Wittgenstein) yet we have private thinking!”* So, it seems, you feel as if you were dealing with something that Wittgenstein dismissed – whatever can be described as “private” is not to be dealt with by (serious) researchers. But my reading of Wittgenstein is different. I think he was talking about a different kind of privacy, and only in a particular context. It is my impression that when you say “private thinking”, you simply say that is inaccessible to an observer. When Wittgenstein denied the possibility of private language, he was presenting his theory of the emergence of languages. What he tried to say was that nobody can have a language for one’s own use and prior to his or her being exposed to communication with others. In this, his stance reminds that of Vygotsky and is directly opposed to that of Piaget. The latter claimed that people are born with an “autistic” (their own, private) language and only latter they learn “social” languages, such as English or Hebrew – languages created for inter-personal communication. Vygotsky claimed that this contradicts the very idea of language. Historically, he claimed, languages could only emerge from the human need to communicate with one another. And this is what Wittgenstein seems to be trying to say: people do not have any need for private languages as they do not even try to self communicate before they have mastered a “social” language. But please note that once you speak in, say, English, it is not impossible to create, an all new one. One can do it completely on one’s own - see Esperanto invented by one individual, Zamenhof. Theoretically, it would be also possible for the inventor to keep this language for himself, thus turning it into his or her private language. Wittgenstein, of course, knew that much while protesting against the idea of private language. As I said, this was his way of describing his vision of the origin and function of language. In this way, he also protested against the idea that each one of us has a private language in which one communicates to oneself her inner (private!) experiences.

So just to return to your query, I don’t think you are using the term *private* the way Wittgenstein did in the context of language. What you seem to me worried about is the inaccessibility of this experience to others and the fact that some aspects of these experiences are not easy to communicate to others (describe in language) even by the experiencers themselves. This inaccessibility is the reason why behaviorists dismissed thinking (mind) as an object of scientific inquiry. But Wittgenstein was not a behaviorist (well, his late doctrine is sometimes described as “logical behaviorism”; this, however, is something different) and, on the contrary, his ideas implied that, at least in theory, we should be able to investigate (thus theorize) our experiences. Here is the argument, as I presented it in my 2008 book:

Although we must renounce the hope for a neat, ultimate (monological) theory of human actions, we do seem to have a chance for creating disobjectified discourses in which the unmanageable complexity of human processes would be describable in a useful, helpful way. Wittgenstein’s argument went, more or less, as follows: To disobjectify the discourse on human doings, one needs to define the keywords of this discourse in operational terms. This means specifying the use of words such as *thinking*, *understanding*, or *feeling* without reference to “private” entities. And if one doubts that such operative defining is, indeed, possible, Wittgenstein offers a simple argument: If we are able to communicate about thinking (or pain, or any other process traditionally considered as inherently private) in our daily lives, there must be *public* criteria for identifying this phenomenon in its multiple manifestations. If so, there is no reason why we should not be able to use these very same public criteria, alas in a more explicit and systematic way, while trying to tell new, more insightful stories about thinking. All one has to do to operationalize a discourse is to observe how people use words and to tease out rules according to which they do so. When Wittgenstein defined *meaning* as “the use of the word in language,” he both instantiated and described this simple idea of operationalization.

Well, I am sorry if I said much more than you asked for. I also regret if, once again, I misinterpreted your question. If I did, I apologize again. I must admit that I enjoy this discussion even if you don’t 😊

But even if you have enough of this conversation, may I end with a request? And the request is this: if I misinterpreted your question again, could you, please, make another try to explain it to me ? More specifically, what is it that you wish to attain and what are the obstacles that hinder your efforts?

Warmly,

Anna

**From:** Ernest, Paul [<mailto:P.Ernest@exeter.ac.uk>]   
**Sent:** Saturday, April 15, 2017 3:20 PM  
**To:** Anna Sfard

*Dear Anna*

*This is very interesting, and you write very coherently and precisely. I will reply at length later - currently on a train!*

*I have an idea. I would like to collect up this dialogue when we are finished (if ever!) and publish it as a dialogue under our names! Of course you will get a final refusal when I have collected it up!*

Best wishes

*Paul*

**From:** Anna Sfard <[sfard@netvision.net.il](mailto:sfard@netvision.net.il)>  
**Sent:** 16 April 2017 15:42  
**To:** Ernest, Paul

Great idea, Paul. Will be really fun to do this.

Anna

**From:** Ernest, Paul

**Sent:** Saturday, April 22, 2017 6:09 PM  
**To:** Anna Sfard

Dear Anna

I appreciate your extended expositions and explanations. So I owe it to you to try to make my question clearer – and perhaps I need more words than I used previously. Like you I am very impressed by Wittgenstein’s ground breaking philosophy which rejects many of the traditional errors of philosophy. Like you I am committed to a dialogical model of knowing and knowledge and indeed explicitly use conversation as the epistemological unit in my 1991 and 1998 books (Ernest 1991, 1998). I see conversation – dialogue between two or more persons – participating in one or more language games embedded in social practices or contexts (what Wittgenstein terms forms of life) as the basis for communication, knowledge, mathematics and teaching and learning. (Currently I am looking at extending my account to make conversation also the basis for ontology and ethics).

Now I accept that we (persons) appropriate conversation and it becomes a significant part of our thought. As you say above, about the “thinking of another. [This is] defined as a (potentially multimodal) conversation a person has with herself”. I’m glad you say it is multimodal since we can converse using gestures, objects and other props as well as our oral linguistic utterances. But what we cannot communicate directly is our sensory and perceptual experiences, which, following Vygotsky, also make up our thinking. Nor our feelings, emotions, etc., as we experience them, which also colour our thoughts (and of course our spoken language – but differently). I can accept that these things (sensory-based and emotional experiences) are woven into what we call our conversations with our selves but supplemented in this way thinking is more than or different from social dialogue with others.

Vygotsky argues that the contents of our mind are not structured the way our speech is. When I think before I speak my thoughts are more limited – concept connections and associations - and speaking is (that is, can be) articulating these connections in more refined, specific and distinct ways than the looser conceptual connections I intuited. In what ways are my thinking conversational? There may be a question and answer structure to my thinking. Indeed every thought might be seen as a response to the previous thought. Furthermore, I would not have the words and concepts that I think with if I was not a fluent language speaker. Without having learned language I would be like most higher animals – interpreting perceptions of the world, experiencing feelings and emotions and acting on these – but unable to reason abstractly in my thought and speech.

Inner speech is not the interior aspect of external speech – it is a function in itself. It still remains speech, i.e., thought connected with words. But while in external speech thought is embodied in words, in inner speech words die as they bring forth thought. Inner speech is to a large extent thinking in pure meanings. It is a dynamic, shifting, unstable thing, fluttering between word and thought, the two more or less stable, more or less firmly delineated components of verbal thought. Its true nature and place can be understood only after examining the next plane of verbal thought the one still more inward than inner speech. (Vygotsky 1934, Section 7. Part VI)

(Insertion: Rereading this I anticipate an objection. If meaning is largely use, how can words elicit meanings, as in “Inner speech is to a large extent thinking in pure meanings.” Below I mention Brandom’s (2000) inferentialist theory of meaning as inferential associations – so words can trigger inferential associations – i.e., meanings, which are a conceptual type of use.)

One of Vygotsky’s great insights is that structures and functions are not static. Drawing on the dialectic tradition of Hegel and Marx, Vygotsky is very clear that our thinking and speaking change and develop with experience and use. The character (I almost wrote ‘nature’ but I am trying to avoid essentialism) of speaking and thinking change both quantitively and qualitatively throughout human life, especially so in the early years. As he says “Only a historical theory of inner speech can deal with this immense and complex problem.” (Vygotsky 1934, Section 7). I believe the later Wittgenstein was still stuck in the philosophical tradition that speaks (writes) in the timeless present and fails to account for the essential role of growth and change. There are places where he allows for change. For example, to paraphrase from memory, he says that every new proof of a theorem changes the meaning of the theorem. In my interpretation this entails a holistic notion of the meaning of theorems (and other propositions) and that this meaning whole is growing and not static. If much of meaning is given by usage, as you extend the instances of use you extend the meaning.

Now you are quite keen to say that speech and thought are of the same stuff, and that methodologically both can be analysed in the same way. I am not so sure. Methodologically there is (at least) one important difference – speech is directly observable whereas thought is not – it must be inferred from observed evidence. To reject this distinction you must reject the Vygotskian account and say that thought is nothing more than conversation one has with oneself. That is, thought is verbal (extended multimodally?) and the problem is to establish what the narrative is rather than to be concerned with the diffuse extra linguistic character of thought – how it differs from narrative, as well as its meaning content.

Now we come to meaning. I accept what Wittgenstein says, namely much of meaning is given by use. He allows for three other sources of meaning – custom, rule following, and physiognomic meaning (Finch 1995), that I ignore for the present. Both Wittgenstein and Vygotsky see language and signs – bearers of meaning – as tools that we use in acting in the world. I think they fail to distinguish language adequately from other tools (objects of use to achieve our goals). Here I prefer to follow Robert Brandom’s (2000) inferentialist account of meaning. Thus for Brandom, knowing a concept is to be able to use it in the game of giving and asking for reasons. Thus, the meaning of words is (largely given by) their use, but it is a central aspect of use, namely the nexus of inferential connections with other words.

I need to hedge this more carefully. In different forms of life there will be different inferentially connected central corpuses of words. Furthermore, the words must be interrelated via sentences/propositions in order to connect inferentially. In addition, there is an important distinction between the structure of overall connections (like De Saussure’s *Langue*) and the actual historically enacted connections made over the course of a given segment of time (analogous to De Saussure’s *Parole*). In reality, only the latter exists! The former (*Langue*) is a theoretical structure sitting on the latter (*Parole*).

But the key point is that words, concepts, sentences, propositions, etc., are tools in a special class of their own (conceptual), unlike other tools (nails, hammers, cars, computers). One special feature of humans is that we are inferential and live in a conceptual universe. By this I mean that we interpret all our experiences through a web of interconnected concepts. My interpretation of inference goes beyond deductive logic. Here is a provisional sample list of inference types that I wrote for another purpose that we use in teaching and learning mathematics, listed in approximate order of increasing formality:

* Copying example, following pattern or rule
* Solving problem using heuristics, reasoning from representation (table, graph, equation, diagram, etc.)
* Reasoning from observations, results, conjectures
* Reasoning by analogy, metaphorically, or using a model (concrete or abstract) to illustrate, represent or justify
* Arguing from general to the particular (specializing)
* Inductive and abductive reasoning from the particular to the general (induction, abstracting)
* Informal deduction using principles of logical inference
* Formal deductive inferences from axioms using rules of deduction or other systems of deduction

This can doubtless be refined and improved – my point is to include more tacit or informal inference types beyond logical inferences. [I also observe that in thought and conversation we use non-inferential links between meanings, words, utterances, sentences including metonymic links, free associations that we can characterise as involving changes of meaning.]

So now I come back to my question. From a dialogical perspective how can we theorize thinking. To satisfy me the account must accommodate the phenomenology of thinking – my subjective experience of thinking. This includes my attention to perceptions of the world, attention to others’ speech. Also there is my attention to my mental space (to use a metaphor) when, among other things, I lie in bed with my eyes shut thinking about theories of concepts, language, etc.; or thinking about a mathematical problem and trying out imagined solutions. These are elements of imagination, which may involve visualisation and conceptual thought. I haven’t explained this very well, but to me it is not reducible to conversation with myself. Or maybe it is, provided I use an expanded multimodal language which has integrated images, emotions, purposes, associations. But then it differs from spoken language. Which brings us by a commodius vicus of recirculation back to my starting concerns!

Warm regards

Paul

**From:** Anna Sfard

**Sent:** Saturday, May 8, 2017 10:55 AM  
**To:** Ernest, Paul [mailto:P.Ernest@exeter.ac.uk]

Hello again, Paul. Hope you’re enjoying the Spanish sun, paella and wine.

In response to your thought-provoking email, I now will try to do two things: First, I will react to your description of where the two of us are coming from – of your own basic position, of your vision of mine, and of your take on our similarities and differences. In the second part, I will address directly the question that is the primary reason for this conversation: How to make sense of what we cannot speak about?

Let’s speak about our similarities and differences. While first reading your last email, I was impressed by the apparent closeness of your positions. We are really kindred spirits, I thought (I always had this soft spot for Anne of Green Gables and her ways of narrating reality). Just consider the fact that we explain thinking in similar terms: my keyword is *discourse* and yours – *conversation*. Many would say that discourse and conversation two different words for the same thing. Yet on the second though, it may not be so. Our similarities appear more striking than the disparities only if you are looking from afar, compromising the resolution of the image. And then, when you take a closer look and go into finer details.....

But wait, don’t let me to get ahead of myself. What I tried to say is that it is crucial for our further communication to clarify our respective uses of those words around which this dialogue is going to evolve. As you may have already noticed, I’m a serial definer. You may see it as a weakness, and it may be annoying, but it is not without a good reason. After all, if we don’t use a word in the same way, misunderstandings are inevitable. What’s even worse, lacking clarity with regard to words, we may end up with a meta-misunderstanding – misunderstanding about the misunderstanding. This is what happens when unacknowledged disagreements about word uses are mistaken for disagreements about facts. [As an aside: I readily agree that each one of us has the right to using words however she or he wants. Operationalizing our own uses of words does not mean the wish to force others into using the words our way – it is only making clear to the other what one means while uttering these words. We may both be philosophically minded, and this debate surely sounds quite philosophical, but we are educational researchers and an obligation comes with this nobility: we are supposed to help people in attaining the goals that they strive to attain, and learning mathematics is one of them. But we need the words that mediate these efforts to be understood by all of us in the same way. Otherwise, how could we raise this tower of Babel?]

I do suspect that we may be using differently some seemingly identical parts of our respective vocabularies. Take this passage from your email:

*Now you are quite keen to say that speech and thought are of the same stuff, and that methodologically both can be analysed in the same way. I am not so sure. Methodologically there is (at least) one important difference – speech is directly observable whereas thought is not – it must be inferred from observed evidence. To reject this distinction you must reject the Vygotskian account and say that thought is nothing more than conversation one has with oneself. That is, thought is verbal (extended multimodally?) and the problem is to establish what the narrative is rather than to be concerned with the diffuse extra linguistic character of thought – how it differs from narrative, as well as its meaning content.*

See, if by *conversation* you mean what I mean by *discourse,* I cannot understand what is wrong with the claim that “thought is nothing more than conversation one has with oneself” (I have also some difficulty following the argument that leads you to this conclusion, but let’s skip this for a moment, because whether the argument works or not, I readily accept this attribution, provided your *conversation* and my *discourse* are synonyms). Indeed, if you equate *conversation* with (what I call) *discourse,* the claim “thought is nothing more than conversation with oneself”, is exactly what I want to say. I am neither ashamed of this nor do I think that it contradicts Vygotsky! The only explanation I have for your evident objection to equating thinking with conversation is that for you, conversation is only a matter of speech, thus of words. In this case, however, my *discourse* is different from your *conversation*. “My” discourse comprises all that goes into the process of communicating, and this includes activities with words, of course, but also all those unspeakables, untouchables and invisibles that we call “private/inner experience” and that give us so much conceptual headache. Sometimes, discourse (communication) may be devoid of words altogether. As I see it, when you mention conversation, you only consider the publicly accessible elements of communication. Indeed, I managed to account for your evident objection to the claim “thinking is a conversation” only when I said to myself that for you, thinking is made of a different stuff than conversation – the two belong to different ontological categories.

Thus, my feeling is that you embrace dualism. Whether knowingly or just by adopting the centuries long tradition of talking about thinking in terms that imply thinking/communicating dichotomy, you draw a clear *ontological* line between the public and the private, the inner and the external. Indeed, I read you as saying that these are different species – see the argument you present in the passage above: it can hold only if one accepts such ontological disparity. You say: “*speech is directly observable whereas thought is not – it must be inferred from observed evidence.”* But note that also inner speech is not observable and must be inferred! So, it cannot really be the issue of visibility that bothers you. As I said, my hunch is that your real worry stems from your vision of private and the public experiences as made of a different stuff.

[And just to make clear how ontological categorization is different from the distinction between visible and invisible: all that matters for the sake of much of our present debate is whether the process we are talking about, be them inner or external, private or public, are material or not. Physicist do not dismiss the idea of black hole as a material phenomenon only because they will never be able to “look it in the eye”, and they do not object to the materiality of elementary particles just because they are too small to be observed directly (note that when you’re observing a kid solving a problem, this is exactly the kind of thing you are doing: observing her self-communication, a.k.a. thinking, alas indirectly). That the interpersonal and intrapersonal communication have different mechanism, there is no doubt. But this does not justify the conclusion about the *ontological* duality – about these two being of different stuff, the bodily and the mental.]

To be on the safe side, let me counter your argument in yet another way, namely by objecting to its conclusion. “*That is, thought is verbal”,* you said, presenting it as my vision of things*.* But for me thinking, like communication, is not just speech, and in many cases, it is not speech at all. I once made the drawing in Fig. 1 (Sfard 2008) to explain the difference between dualists and non-dualist visions of the mutual relations between thinking and speaking.

And now, to the second part – to your question of “how to theorize all this” – how to theorize the unspeakable. At this point, I can only respond with another question: What do you mean by *theorizing*? Indeed, I am asking you to define the expression *theorizing X*, what else?

**Questions about relations between thinking and speech**

**Q1** (***Separateness***): Are thinking and speaking two distinct processes all along the way, that is, is there a separate process of thinking that runs in parallel to speaking even in the case the latter process is inner?

**Q2** (***Extent***): Is there thinking that does not involve speech at all?

**Possible answers**

|  |  |  |  |
| --- | --- | --- | --- |
|  | 1 the same extent  speaking and thinking are always concomitant | 2 different extent | |
| **a.** there is thinking without speaking but no speaking without thinking | **b.** there is thinking without speaking and speaking without thinking |
| 1 dualist  thinking and speaking are separate processes | 1.1 thinking and speaking, although not the same, always run in parallel | 1.2a | 1.2b |
| 2 non-dualist  whenever there are both speaking and thinking, they coincide (are the same process) | 2.1 thinking and speaking are exactly the same - a single process | 2.2a | 2.2b |

|  |  |  |
| --- | --- | --- |
| The stripped bar |  | represents the process of *speaking* (either external or internal) |
| the shaded bar |  | represents the process of *thinking*. |
| The horizontal dimension represents time; points located on the same vertical line are concomitant. | | |

Figure 1: Mutual relations between speaking and thinking

I already proposed in passing that our present X be called “the unspeakable”. I believe we are in accord here? True, you said that you wish to theorize *thinking*, not just unspeakable, but this is what I thought I did while equating thinking with self-communicating and you responded that I might not have done a proper job. “*To satisfy me”*, you said, *“the account must accommodate the phenomenology of thinking – my subjective experience of thinking.“* So, in your opinion, the subjective experience of thinking is not yet there when thinking is defined as a form of communicating. From this I deduce that as far as you are concerned, my theory of thinking is missing – can we agree on that? – an account of the unspeakable. Theorize it is our remaining job.

Now, in my view, theorizing X can be defined as telling a useful, consistent, exhaustive (as much as possible) story about X. Indeed, theorizing is the most advanced type of making sense of our experiences, whether private or public, and people do this by concocting stories that impose order and structure on reality, turning our experience into a network of relations. Such stories are *useful* if they also help us to improve our practices (and let’s leave for later the debate of what counts as improvement and who is to tell). According to this definition, theorizing the unspeakable means *constructing a useful consistent story of the unspeakable*. But here is a difficulty: If theorizing unspeakable is telling stories about it, your undertaking goes against Wittgenstein’s famous advice encapsulated in the bottom line of *Tractatus*: “What we cannot speak about we must pass over in silence”. So, my first query to you is this:

**Q1. What makes you think that you may ever be successful?**

And to another query. Obviously, more than one consistent useful story can usually be told about the same set of phenomena. What story do you want to tell? Which questions about our private experiences is this story supposed to answer? In fact, you’ve already began telling me one such story in your last email: “*I lie in bed with my eyes shut thinking about theories of concepts, language, etc.; or thinking about a mathematical problem and trying out imagined solutions.*”. Do you see this brief story as already an attempt at theorizing? If not, what is it that is missing?

One way to indicate which story may be seen as a theory is to say what questions it needs to answer. So, the second query I wish to pose to you is this:

**Q2. What questions about your private experiences do you wish your theory to answer? Tag those that belong to your list: *What are these private experiences? How can I tell whether the person who tells me about her private experiences reports them faithfully? Why do these feelings appear?* Any other?**

But, wait, having said all this and formulated all these queries, I am suddenly aware that, in fact, we may already be done with exactly what you are asking for. Let me repeat: you say, “*To satisfy me the account must accommodate the phenomenology of thinking – my subjective experience of thinking“,* but as far as I am concerned, the commognitive account of thinking does accommodate the subjective experiences underlying thinking. I’ll explain this in a second, but whether I have a chance to convince you depends, of course, on how you interpret the word *accommodate*. For me, *to accommodate the unspeakable* means taking into account the existence of these subjective experiences *always*, and especially while analyzing human actions and speculating on their sources and effects.

Let me add clarity to this idea with the help of one example: of how *the experience of understanding* is being dealt with in my work (see e.g. Sfard 2008). My solution is that rather than asking “Did the student understand?” – something that discloses the intention to objectify the inherently private experience of understanding (what else would it mean to find a reliable shared vocabulary for talking about it?) – I am asking, “What does a person say about her experience and how is her story related to the ways she behaves and does things?” Thus, I make a massive use of people’s *first-person* descriptions of their experiences. For instance, I always pay utmost attention to what participants of my studies tell me about how they feel, about what they thought while doing this or that, and about how and whether they understood something. One thing I am *not* trying to do is to build *third-person* descriptions – researcher’s descriptions – of their experiences. If I did, it would be a bit like saying “The person said she felt\visualized\.... like this, but the truth is, she felt like that”. The advantage of the focus on first-person reports is that I am dealing here with two public phenomena: stories that people tell and things that people do. My job as researcher is to observe these two and try tell how they are related.

And here is my method: While studying understanding and learning, I rely on two sources:

(a) *the learner’s reports*: the learner’s first person accounts of her experience of understanding; and

(b) *my first-person story about my own experiences of understanding*, about eh experience that causes me to use the words, “I understand” or “I don’t understand”, and lets me empathize with other people who say the same.

You may say that you are not satisfied – that these two types of data are not enough. You may claim that you need also a reliable third-person account of the learner’s understanding. But what is it that makes this third person account necessary? Well, the teacher needs them, you may say. And the fact is that teachers *are* in the habit of giving third-person accounts of their students’ understanding. I have two things to say about this. First, these accounts are nothing else than intuitively combined first-person accounts of types (a) and (b): when I say “A understands”, all I do is attributing to A the feelings, sensations and other experiences that I go through myself when I talk and behave the way A does. Second, only too often, thus concocted third-person accounts turn out to be at odds with the first-person accounts of the experiencer. If so, they are not of much practical use.

To sum up, I am not trying to be an observer of what is happening in the head or heart of a person who says “I understand” (or “I love” or “I am sad” or “I am in pain”). Instead, I am trying to figure out how her saying these words and showing all those emotional expressions, whether verbal of bodily, is related to the ways she is doing things. To find these relations, I am alternating between the perspectives of insider and outsider to the colloquial discourse on understanding. It is the insider’s perspective – the perspective of a person who has first-hand familiarity with the experience that prompts one to say “I understand” - that gives me the intuitions what connections may be found. In other words, my ability to empathize is my guide in asking questions and raising conjectures about possible ties between one’s experiences and her actions. I need outsider’s perspective to control m y intuitions and guard me against writing too much into what I see. It also protects me from noticing what is not really there.

Naturally, my last query to you, for today, is this:

**Q3. Is what I said the kind of theorizing of the unspeakable that satisfies you? Does is sufficiently accommodate the phenomenology of thinking?**

Well, this piece is much longer than intended – and I have not even touched upon Brandom and what you take from his work. On the latter, let me only say this: I am no expert on Brandom, but at the first sight what he says is not unlike what Wittgenstein said. Or, to put it differently, Brandom seems to be zooming into what Wittgenstein outlined, and he does it in the attempt to produce a higher resolution version of what the Wittgenstein has sketched in broad strokes. I will be learning about his work as we go, I suppose. Thank you for sending me his book.

Warmly from Melbourne,

Anna

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**From: Ernest, Paul**

**Tue 04/07, 10:45**

**To: Anna Sfard <sfard@netvision.net.il>**

*Dear Anna*

*Thanks for your kind and collegial approach to time delays etc*

*Here is my latest instalment in our dialogue - sorry it's a bit long - I got carried away!*

Best wishes

*Paul*

**Meaning as Use**

Like you I follow Wittgenstein's idea that the meaning of a word is in many cases given by its use. However this needs disambiguation, for "use" has multiple meanings. The particular use which I make when I utter the word "red" or "addition", say, at a specific event is one such enactment of meaning. But the system of use or usage has another meaning. This includes a systematic theory of usage that describes past correct usages and potentially includes future correct uses - or at least the rules that will guide them. De Saussure made this point when distinguishing *Parole* (utterances of spoken language) from *Langue* (the system of language). Now of course the spoken utterance meaning of use comes first. There is a history (we all have histories) of language uses, and we all have a set of memories of instances of language uses - our own and others. In addition, these memories will include the corrections we have received, observed or given, which have shaped our capacities for spoken language. Well, in fact, we may not remember all such corrections but our linguistic know-how will have been shaped by such instances of correction and correct usage, from childhood on. On top of this we may have explicit ideas of when and how to use such words in ways that use or enact their meanings correctly. We may also have an explicit theory about how such meanings interlock with other word meanings. These theories will either be self developed, learned from others, or resulting from a combination of the two. Use in this systematic, theoretical sense is secondary to specific instances of participation in conversations and making or hearing utterances. (Parole precedes Langue.) I should add that many persons may not have explicit or full theories of word use, but have the capacity to make and understand meanings from word utterances.

So what's the point of this in the present context? Well the meaning of a word as given by a specific utterance is only partial. We need to have a whole pattern of usage to give a better indication of meaning. This pattern might only be a set of utterances, or it can be such a set and more, such as an explicit theory or a tacit set of guidelines or intuitions, as I described above.

Drawing on Wittgenstein's idea of meaning as (mostly) given by use you conclude that meaning is neither a thing in the world nor a private entity in one’s mind, but is an aspect of human discursive activity and, as such is public and fully investigable. While I mostly agree with you, I contend there is a difference between meaning as a specific instance of use (an utterance) and meaning as usage (a set of utterances together with tacit or other rules or capacities). Utterances can be observed but the totality of usage cannot. It has a theoretical element. We can observe at best a sample of the set of utterances.

As I said earlier in this discussion, I like to follow Robert Brandom’s (2000) inferentialist account of meaning. Thus for Brandom, knowing a concept is to be able to use it in the game of giving and asking for reasons. Thus, the meaning of words is (largely given by) their use (in language), but it is a central aspect of use, namely the nexus of inferential connections with other words. For Brandom the inferentialist meaning of a word or sentence S is its connections through reasoning with antecedents (reasonings leading to) S and its consequences (reasonings that follow from S). These uses are shown through enacted utterances, but meaning reflects past uttered links and is always open towards the future. So the 'summative meaning' of a word or sentence, if I dare to use this (self-contradictory) term, is an unfinished theoretical activity. This admits intertextuality - linking between texts that treats texts as making up a network, and the dialogical view (after Bakhtin 1994) that there is no text other than intertext. Whenever a text comes into being it relates to previous texts and in its turn becomes the precursor of subsequent texts. Inferentialism offers one way to understand the dialogicality of the meanings of texts and utterances.

Your main point is that meaning is given by publicly observable utterances. Rather than looking for a hidden simplifying mechanism, we should deal with this complexity in its own terms. These observable phenomena are not “windows” to another, inherently private universe. You go on to make an ontological point that thought (thinking) as a mental dialogue is of the same 'stuff' as a social dialogue between persons. Because I believe in a monist universe, a monist ontology, there is nothing but the dance of matter/energy, so private and public dialogues, like everything else, are made up of nothing but this 'stuff', in some of its boundless forms.

However this ontological similarity (or identity) does not guarantee that the same methods can be used to investigate them. To explore elementary particles, the dynamics and statics of human sized bodies, and the bounds of the known universe requires quite different techniques even though they are all made of the same 'stuff' ontologically. You acknowledge this explicitly. You have argued that speech and thought are of the same stuff, and therefore methodologically both can be analysed in the same way. I disagree and have argued there is an important difference – speech is directly observable whereas thought is not – it must be inferred from observed evidence. I do not say we should give up trying to investigate it because it is an inherently private phenomenon. We just need different methods.

We can make a recording of student speaking as they 'do' mathematics in a group, solving a joint problem. That captures the whole dialogue, especially if we also record body language, marks on paper or on a computer screen that accompany verbal dialogue. We can also record a student speaking aloud as they solve a problem on their own. However, there is a difference. The self report is secondary to the actual problem solving process - a meta-narrative about the solving process, unlike the group dialogue which is the solution process. We could also ask the single student what they 'see' or imagine as they solve the problem, as another source of data. But all the evidence for the private narrative will be filtered through the student's own intentions, choices, selections, conceptions and ability to describe verbally a process that is both more and less than purely verbal. Issues of inadvertent misconstrual or deliberate misrepresentation arise which they do not with group dialogue. In Ernest (1997) I asked a student how she solved a simple trigonometry problem A, which she had done on paper in front of me. The account she gave did not actually describe what she had done. Instead it represented a solution to a different problem B. Namely "what is the easiest way to communicate your method of solving problem A to another person?". And just as problem A and B differed, so too the solution to problem B was not the same as that to problem A.

I use the words inner and outer dialogues or narratives to distinguish private thinking as the dialogue the mind has with itself (as Plato described thought) from public dialogue, made up of the spoken out-loud word utterances which are both audible and directed to another. Again I need to add that public dialogues include more than a stream of uttered words. Apart from the fact that other languages are available and I believe equally good, for example British Sign Language, there are also the signs that accompany speech. These include body language, movements, signs on paper or screens, or deployed in the environment, and so on. We can also have dialogue in purely written forms, as in our current dialogue. But my point here is that the use of the inner/outer metaphor for private/public, respectively, need not invalidate my monist ontology. The distinction is not absolute. Sometimes a person will speak to themselves, enacting their inner/private dialogue in the outer/public arena, sometimes without even their own knowledge. To avoid the words inner/outer can lead to cumbersome circumlocutions

**How do speech and thought differ?**

In your last response you offer a nice diagram of dualism/non-dualism and thought/speech. I like the four possible states of non-coincidence and overlap. However your model assumes that mentation can be divided into speech (the process of speaking) and thought (the process of thinking), and then you question the relationship between these; whether coincident in nature - or not. What you want to say is that mental (private) speaking and public speaking are identical. And that if there is a leftover in thought that this is unsayable and inexpressible (I'll return to this later). My argument is that thinking cannot be identical with speaking. For speaking is nothing but public sounds and signs, while the other (thinking) is not necessarily vocalized and made public. They differ in properties therefore they are not identical. Maybe you want to make the weaker claim that they are ontologically the same. I agree, but then in monism everything is made of the same matter / energy in various dances and patterns. Maybe you want to say that, for brevity, what I will call private or public speech have the same origins. I agree with this and have given an extended account of this. But when you say that thinking as private speaking and public speaking are identical I cannot agree. Maybe you mean that the differences exist but are trivial. Well then I need to know what the difference is between the trivial and non-trivial characteristics are, and we are back to square one.

So I come again to my central question. How is thinking different from speaking? I asked if you could either account for this difference or give a theory of thinking that respects the difference. By a theory I mean a general account of some phenomenon that explains what it is, how it works, and predicts some testable outcomes. I am happy to use the term loosely to include some explanatory and hopefully predictive models. This may not be fully rigorous like theories in the physics, because it cannot be reduced to mathematical equations.

Now asking you to explain the difference between thinking and speaking may seem unfair because you deny their difference. Instead you could explain whatever thinking is, how are we able to be creative, imaginative, and to bring visual and other imagery that are not reducible to language into our deliberate and creative thinking. Also, what is internal speech? Do we imagine a string of words as if we were uttering them out loud?

However, through this dialogue with you, being forced to articulate my own ideas, I am now moving towards giving my own account of thinking. You reach the same conclusion - that I should attempt to do this.

To do this, first I need to sketch the genesis of thinking, which I do here. A baby has sense impressions of its world of experiences, probably beginning during its period of gestation. Some of these sense data originate from outside its own body - such as from light (impacting via seeing), from sound (via hearing), from touch (via skin pressure nerve arousal). Some of these experiences originate from within the baby's own body, such as hunger, bodily discomfort, spontaneous emotions. These two sources of experience are deeply interwoven. And the distinction is far from absolute since sensory inputs must be interpreted in both cases. I believe that the baby notices invariants and starts to impose some order, structure or pattern on its experiences giving rise to what Vygotsky calls spontaneous concepts. What such concepts are I cannot say precisely but they may not only include regularities in sensations but also regularities in responses such as movements, vocalizations, etc. Undoubtedly these concepts vary and grow over time, they are not static and need not be constant. The baby is not isolated in this world of experiences, actions and concepts because the baby is involved in preverbal dialogues (reciprocal actions and what we might call signalling) with others, most notably the mother or primary care-giver.

At this stage it is hard to know what the baby's thinking is like. Presumably there will be 'inner' sensations and experiences such as pleasure and discomfort or displeasure, recognition of familiar persons, objects and experiences, associated emotions and feelings, sensory images recalled from memory. There may well be reactions to non familiar persons, objects and experiences, accompanied by emotions such as interest, curiosity or anxiety or fear, such as being startled by sudden loud noises. What the stream of consciousness is like I cannot say, but I expect it will be led by sensory stimuli, whether external or internal in origin.

Now we move to the next stage, although of course this overlaps with the preverbal phase, and ultimately engulfs it, as I shall argue. Other persons, such as the mother, will start to use words with the baby, beginning a verbal dialogue. I ignore intermediary phases such as babbling (in pretend speech), etc. After some exposure to this the baby will start to use words back, mummy, daddy, ball, dog, or whatever. The baby starts to use these words in a regular and recognisable way. At this stage the baby/young child is starting to develop what Vygotsky terms scientific concepts, which would be better termed social or cultural concepts. The use and mastery of language takes quite a long time and during this time the child develops and uses a growing set of linguistic capabilities. Of course this development is triggered by engagement in a growing range of dialogues in different contexts, with different purposes, and with different but overlapping vocabularies.

Somewhat later during childhood, most children also start to learn to read and write and these encounters with written language may also feed into the development of their thinking. This includes written arithmetic and other parts of mathematics. However I won't speculate on the impact of reading and writing on thinking.

A second strand of development concerns attention, which is part of human agency. A baby turns to look at objects or people that interest it, or that move and draw their attention. Of course other sensory stimuli also capture its attention, sounds, touch, smells, tastes, pain, and so on. As the child develops its power of self-directed attention grows and becomes increasingly volitional. In addition to choosing what to attend to in its experiential (perceptual) world, the child can choose and initiate its own activities. It can direct its attention to different activities including toys, games, video, TV programmes and other things. One of the most important things that a child attends to is other humans and dialogue. The child attends to many utterances from others and participates in dialogues.

So now the stage is set for me to propose what thinking is or at least might be. According to Vygotsky the child's spontaneous and scientific (that is, linguistically acquired) concepts meld or at least start to interact and form one inner system of concepts from quite early on.

So private thinking consists of an inner dialogue the person has with themselves. And this will be learned from participation in conversations and discourse with others. But this inner dialogue is not just made up of words - it is supplemented by and may even have elements replaced by visual imagery, memory episodes, feelings (emotions, etc) within the internal stream of consciousness. An associational logic is at play so perceived external persons, objects or events may trigger associations that become contents in the inner dialogue. Thus thinking, the inner dialogue, may be a string or cluster of meanings, concepts or reasonings. This may be prompted by external stimuli, such as conversations/speech from someone else, experiences or events in the world. or may be internally generated, such as when I solve a mathematical problem mentally. The stream of ideas etc that I experience in thought is multimodal and can involve words and associated concepts, imagery both real and imagined, smell and touch impressions, or memories of them, etc. We also have some control over this internal dialogue, we can choose to remember something, direct our attention to some idea, memory, problem, etc. Of course things also come unbidden to our thought, either because of some deep unconscious trigger, or an association that draws our attention aside or onwards.

In what way is this internal stream of consciousness a dialogue? Well, in three ways. First, learning to speak is by means of participation in dialogue and conversation. So languaging is a process driven by public speech, that is words and speech. These evoke meaningful concepts and reasoning responses in us - their content and form is irrevocably tied in with their origin, that is spoken dialogue or conversation. Vygotsky is often interpreted as saying that speech and dialogue become internalized. This is of course a metaphorical rather than a literal description. Children learn to imitate phrases. I expect they can also imagine the sounds of these utterances subvocally, that is solely in the mind. So exposure to speech leads to something like speech in the mind.

Secondly, our streams of thought come in segments. How these are demarcated or segmented I cannot say, but each segment will have a coherent meaning. Each of these thought segments evokes an association or follow on, a response or reply. Thus we follow each thought by its echo or answer, like question and answer, thus exhibiting the dialogue form. Just as in a spoken dialogue, we have choices as to how to choose/make our replies thus steering the conversation. Likewise in our internal dialogue we can choose how to follow on a line of thought. Of course some people with compulsions find it difficult to steer away from a recurrent patterns of thought. Indeed this can happen to any of us if we are stressed by a difficult situation or conflicting or difficult demands, or an unsatisfactory ending to a previous conversation. So all my general claims must be hedged with caveats because less typical events and cases can always occur.

Thirdly, our thinking is dialogical when we are reacting to an artefact - a piece of writing, painting, a performance, or even someone talking including a lecture. The attended to part of the artefact is one voice in the conversation and our reactive or reflective thoughts constitute the second voice. which we may or may not utter in public.

Our internal dialogue can have a variety of functions. It might involve planning something, a solution to a problem, a plan for making something, the development of ideas. This is imagination at work in thought. This may involve all sorts of meanings including concepts, word meanings (associations), visual imagery, practical sequences of actions. However, such planning or creative imagination need not anticipate or take place separately from our activities. For often we can be involved in making something, such as me writing these observations, and not know beyond a hazy idea, if we have that, where our stream of ideas or words - our internal dialogue - is going. Often our next step in the creative process is enacted as the moment arises. It is a choice, what feels like the right choice, the necessary choice, in the moment.

In writing this, I've let the account pour out of me without careful attention to the use of some words, such as inner, concept, meaning etc. In fact I've written that last couple of pages during a plane and train journey, and then immediately after in a hospital waiting room to get a pain in my leg seen to. (It was all okay!). These are not an excuse for sloppy thinking, but I don't want to be held to ransom for some careless word which I can rephrase more carefully or cautiously. and I'd be happy to have that pointed out! It may be that as I generalize this I need to go back and remove hidden individualistic assumptions in this account to let it flourish as a fully dialogical account of thinking. It is hard to overcome the individualism, monologism and other received ideological assumptions that have historically been built into our public discourses.

**Roles and Power**

In any dialogue persons as active agents in that dialogue take on a variety of roles. Two of the most important are speaker and listener. Speaking can involve offering new links that are responses to the previous utterances. Such responses can build on, extend what was previously said. Or they can interrogate and question what was said. Listening can be actively following the narrative and making sense of it through linking utterances with our own concepts and meaning associations. We can follow the flow of a narrative adding our own associations and responses, which we may (or may not) utter audibly or publicly. We can listen critically whereby we interrogate, question or challenge the narrative as we are hearing it. We can make these reactions public or keep the thoughts to ourselves. And we should never forget that we are embodied, not just passive in listening or active in speaking - we are all the while engaged in bodily activities beyond the actions of communicating (vocalizing, facial expressions, arm and bodily movements) for we can also be drinking coffee, walking along a road, or even building a model or material artefact together.

**Dialogical space**

I can now generalize this local account of thinking to offer a fuller account of dialogical space. This is the virtual space in which words, gestures and signs are uttered, perceived and responded to. Dialogic space or spaces are both public and private. A conversation between persons has 'visible' multimodal utterances which are public, but also runs through our private spaces of understanding where we attend to the dialogue and create or conjure up associated meanings, imagery, emotional responses in our reception of the dialogue.

In the following I capitalise the terms I use in Figure 1 below. In listening we have to pay Attention to what is being said, understand it in terms of building the Meaning links to what we know (the network of words and concepts to which we have personal access). Through understanding we take personal Ownership of the meaning links to antecedent and consequent words in our network of reasoning relations. Then, if we have an Expressive impulse we loosely assemble the Idea and express it as our Chosen supplement to the dialogue which we Utter. Every participant in the dialogue does this. Participants also own a set of Rules about how the dialogue should be conducted in terms of participative membership, the appropriate form of contributions, and the conceptual content of contributions. This overall process is illustrated in Figure 1.

*Figure 1: Dialogical space and with its personal corners*

Person 1

Person 3

Personal corner 1

Personal corner 3

DIALOGICAL SPACE \* Utterances

\* Shared

Rules

**Utters**

**Listens**

Chosen supplement Attention

Idea Meaning links

Expressive impulse **←** Ownership

Person 2

Thus, in addition to exploring and developing the ideas under discussion (the content of the dialogue) participants' contributions can also be utterances that are about regulating or policing the dialogue based on rules that should reflect shared values and democratic principles. For example, in a dialogue between friends and colleagues one or more contributors may intervene about imbalances in contributions, for example, some participant speaking too much or another being encouraged to contribute and be attended to. There can also rules-based utterances on the content of the dialogue, which may be commenting on, redirecting or curtailing some contribution to steer the direction or thrust of the dialogue in terms of the content and concepts discussed. There are power differentials between contributors in most dialogues, based on personal force or institutional authorization. Table 1 lists some sample types of conversation with and the relative power of the participants indicated.

*Table 1: Types of conversation and the relative power of participants*

|  |  |  |
| --- | --- | --- |
| **Type of conversation** | **More powerful participants** | **Less powerful participants** |
| Collaborative research project | Power moves around group | Working researchers less powerful if there is principal researcher |
| Informal Conversation between colleagues | Power moves around group unless power hierarchy has been established | Power moves around group unless power hierarchy has been established |
| School maths class | Teacher - directs teaching and the learning activities | Students - follows teacher instructions and rules for participation |
| School maths examination | Examiners | Students (examinees) |
| University seminar | Visiting lecturer | Audience - but audience can take some power in the questions slot |
| Journal editorial board | Editor, referees | Author |

Thus, as in these examples, there are many formally distinguishable forms of discourse / dialogue / conversation with conventional roles and allocated powers. As educationists, mathematicians and citizens we not only encounter but are used to many of them.

**The Unsayable**

In your text you refer to the unspeakable, and quote Wittgenstein’s famous advice in the bottom line of Tractatus: “What we cannot speak about we must pass over in silence”. First, I believe that many citations of this dictum are misuses. As you have acknowledged, the Tractatus was a project to erect the ultimate picture theory of meaning. Virtually everything, according to this theory, corresponds to some disposition of matter in the world. The meaning of a term (noun) is what it names, the meaning of a sentence is a state of affairs that holds in the world. This is because one founding motive of the logical positivists was to move to cleanse meanings and eliminate metaphysics, ethics, etc from scientific language. Wittgenstein was a founding father of Logical Positivism in Vienna in the 1920s, a movement which promulgated this 'meaning cleansing' programme. One or two decades later, Wittgenstein and many others including ourselves, rejected this picture theory in favour of the "meaning is use" doctrine which we have already laid out clearly in this discussion. So what must be passed over in silence (note this is a normative sentence and hence meaningless in the terms of the criteria of the Logical Positivists) is much stuff that we now want to allow to be discussed.

This does not mean that there is not stuff that is unsayable. Table 2 contrasts the possible combinations of the known and the sayable

*Table 2: The Known/unknown and the Sayable/unsayable[[2]](#footnote-2)*

|  |  |  |
| --- | --- | --- |
|  | Sayable | Unsayable |
| Known | Explicit knowledge - the Known sayable | Implicit knowledge or 'know how' - Known unsayable |
| Unknown | Freudian slips and other unanticipated and perhaps non-understood expostulations or utterances - Unknown sayable | Unknown unsayable - any such contents are inaccessible unless they move into one of the other quadrants |

Looking at the four quadrants in Table 2 in turn, obviously we have the Known and Sayable. This includes explicit knowledge, beliefs and information, such as this discussion.

Secondly there is the Known unsayable, including implicit knowledge or 'know how'. We may be able to put some of this into words, but we cannot express it completely in explicit verbal form. It can never be fully translated into explicit statements.

Thirdly there is the Unknown sayable. This can include Freudian slips and other unanticipated and perhaps non-understood expostulations or utterances; speaking with tongues, babbling, etc. Such utterances may be understood later (i.e., 'known') on reflection and analysis.

Fourth and lastly there is the unknown unsayable. Any such contents are definitionally inaccessible while they remain unknown and unsayable. They can only be known if they are transformed and appear in another one of the other quadrants, and even then their origin in the fourth quadrant cannot be known.

In the present context the second quadrant is the important one, the known unsayable. Some of our knowledge is tacit, implicit 'know how' which we can apply but not express completely and fully in explicit and verbal form. In mathematics at all levels many methods are known implicitly and can be applied in particular situations without being fully statable. This would fit with one of Wittgenstein's other types of meaning: following a rule. Many complex rules are only known and applied through instances. Kuhn (1970) makes this point when he writes about paradigms (in the narrow sense) as problem solution exemplars. So the unspeakable and unsayable matter in our thinking and doing mathematics, as well as elsewhere, and should not be written off.

I hope I have now answered your three questions indirectly if not directly.

**Your method**

You come to an end by suggesting your sources of knowledge about understanding are twofold.

And here is my method: While studying understanding and learning, I rely on two sources:

(a) *the learner’s reports*: the learner’s first person accounts of her experience of understanding; and

(b) *my first-person story about my own experiences of understanding*, about eh experience that causes me to use the words, “I understand” or “I don’t understand”, and lets me empathize with other people who say the same.

I think this is a bit restricted. These may be your two data sources, although I am surprised you don't include your dialogue with the subject as a third source. But in addition there is your whole explicit and implicit theoretical framework and set of assumptions about personhood, narrative/discourse, as well as epistemology and ontology. What I am talking about is trying to make our theoretical framework and assumptions explicit - or at least as explicit as possible.

Finally, I agree with your remarks on Brandom. Characterising meaning in terms of specific conceptual uses - giving and asking for reasons and word meanings as given through reasoning connections - is taking a central feature of Wittgenstein's meaning as use doctrine and elaborating it. I think it fits well.

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**From:** Anna Sfard

**Sent:** Tuesday, November 7, 2017 03:55 PM  
**To:** Ernest, Paul [mailto:P.Ernest@exeter.ac.uk]

I’m looking at this long thread, Paul, and can’t help noticing what you just noticed: with time, our respective moves have been becoming longer and longer, and also more distant in time from one another. This worries me a bit, and not just because the spontaneity is gone, and with it, some of the fun is gone as well. As an email exchange, our conversation was supposed to be a ping-pong of quick serves and rapid responses. Instead, it became more like a game of squash, with one player’s moves only marginally dependent on those of the other, if at all. And I am wondering: Isn’t this a sign that our dialogue turns into a series of monologues, and if so, shouldn’t we pause and ask ourselves how pleased and satisfied we are with what we’ve been doing?

Such meta-reflection seems particularly appropriate considering the frequent appearance of the word *dialogue* in your latest email. Dialogue, after all, is what we seem to have been engaging in while exchanging these emails. Since both of us have declared, in one way or another, our utmost reliance on dialogue in theorizing about human thinking, it is only natural to illustrate the things we say with the most obvious, most immediately accessible, example: our own exchange. I thus decided that in this email, which will be my last contribution at this stage, I will relocate to the meta-level. In this meta-conversation, I will be asking – but not necessarily answering – the question of how well our own dialogical activity exemplifies our respective visions of how things should be.

At this point you may protest, saying that the use of the word *should* is at odds with the spirit of our exchange: we have been acting as researchers and as such, spoke about how things *are* rather than about how they *should* be. Indeed, ours has been the language of factual statements, not of proposals or requirements. Sentences such as “the baby is involved in preverbal dialogues” or “Now we move to the next stage, .... beginning of a verbal dialogue”, taken at random from your latest email, are representative of our genre. And yet, isn’t this scientific neutrality but an illusion? The term *dialogue* has been central to the work of many 20th century thinkers, who, while using the word, engaged in anything but an impassionate portrayal of reality. Their recourse to the idea of dialogue has been motivated by deep social and moral sensitivities. Renouncing any pretense to impartiality, they declared their “passionate longing for dialogue” (Buber, 1923/1958), spoke about dialogue as “conquest of the world for the liberation of humankind” (Freire, 1970/2005, p. 2), and claimed that “If the structure does not permit dialogue, the structure must be changed” (Buber, 1923/1958). Ever since these declarations, the mere utterance of the word *dialogue* is likely to be interpreted – if not by the addresser herself, then by her addressees – as signaling an endorsement of whatever values appear to be implicated in these statements. In short, the term *dialogue* in one’s mouth renders the utterer an identity of prescriber rather than describer, of ideologist much more than scientist, of activist as opposed to a mere observer.

You may object again, stating that all the above quotations employ *dialogue* in a metaphorical sense, and this is quite different from how this term functions in your narrative on thinking. I readily admit that the word is polysemous – which of our keywords isn’t? – and that different authors, without recognizing, are using it in different ways. I will say more about this in a minute. And yet, I also ask you to keep in mind that uttering certain words may sometimes allow us accomplishing several communicational tasks at the same time. Contributing to a story about a certain aspect of reality – in our case, to the story of human thinking – is only one, probably the most obvious, of these tasks. Another one, often implemented without explicit intention, and thus unlikely to involve rational choices, is telling a story about one’s own values, preferences and commitments. In this case, some of the discourses strongly associated with the word that has been uttered tend to overwrite the discourse within which this word has now been uttered. Whether the utterer wants it or not, she will now be seen as taking upon herself commitments that come with participation in those other discourses.

Having said all this, I readily jump into the discourse you have evoked by your “dialogic turn” and, as a participant, I endorse willingly the legacy of the thinkers who embraced the idea of dialogue as an epitome of humanness. It is because of these thinkers and their teachings, and due to some of the things you said in your latest email, Paul, that I was disquieted by the realization that our dialogue has been turning into a series of soliloquies.To explain why, I need to take a closer look at, first, the idea of dialogue as such, and then at the question of why this idea was central to the task we took upon ourselves while engaging in this exchange.

Before elaborating on the concept of dialogue – the undertaking in which I will be trying to get some inspiration from the relevant literature – let me comment on your own use of the world. In your last piece, you say things like “(thinking) as a mental dialogue”, and after you state “now the stage is set for me to propose what thinking is or at least might be”, you declare that “private thinking consists of an inner dialogue the person has with themselves”. Judging from the context, this definition of thinking is offered as a counterproposal to mine, according to which thinking is a form of communication. But are these two conceptualizations really different? Is anything added by the change of words, from communication to dialogue? Well, as long as we remain focused on the question “What is thinking?”, I can see no difference. I’m saying this in spite of your numerous “buts” (as in “But this inner dialogue is not just made up of words“) and of the fact that your decision to present your position in detail signals the intention to say something new. Yes, in spite of all this, I believe that when used in the definition of thinking, your *dialogue* is identical with my *communication.* While saying this, I am also asking, and not for the first time 😊: Please revise your conviction that I view thinking “as private speaking”. As I repeatedly tried to make clear, I opted for the word *communication* (which signals multimodality, by definition), rather than *speaking* (which is verbal, by definition) exactly to forestall the impression that I’m equating thinking with (inner) speaking.

So, as far as the story of thinking is concerned, your change of vocabulary does not seem, to me at least, to matter. It is only when one realizes that the word *dialogue* is a secret bridge to the discourses of Bakhtin, Gadamer, Buber, and Freire, to name but a few, that one must also agree a new dimension was added. Here, dialogue stops being just any form of exchange – it becomes a conversation *of a certain kind*. It is our capacity for this special form of communication that constitutes the gist of our humanness, with this last word embracing both our biological and cultural uniqueness, and thus indicating much more than the fact of our being a part of a certain well-defined species.

This may be the right moment to take a more thorough look on what exactly the word *dialogue* stands for in the writings of different dialogically-minded philosophers. In the view of the dazzling supply of high-quality, impressively rich descriptions, the task is challenging, to say the least. So, let me just mention some of the proposals. I’ll begin with Gadamer who, to make clear that not just *any* exchange would contribute to our uniquely human, dialogical selves, requires that communication take the form of the “genuine dialogue”, defined as

a process of two people understanding each other. Thus, it is characteristic of every true conversation that each opens himself to the other person, truly accepts his point of view as worthy of consideration and gets inside the other to such extent that he understand not a particular individual but what he says. The thing that has to be grasped is the objective rightness or otherwise of his opinion, so that they can agree with each other on the subject (*Truth & Method*, 1975; p. 347; quoted in Bernstein, 1986, p. 112)

For Bakhtin, a literary theorist interested mainly in the form of communication known as *text*, an exchange or exposition can be called dialogical if it makes references to other texts. Dialogicality, therefore, means making space for many voices, and requiring that these voices never stop posing questions and responding to one another. To put it differently, dialogicality, for Bakhtin, is not just a matter of the multiple presence, but also of a genuine engagement, the ultimate expression of which is that one voice allows itself to be shaped by another. As an aside, Bakhtin refuses to speak about dialogue in the absence of language. Dialogical relations arise and are embedded in words (which does not preclude multimodality, of course), and this is what makes them uniquely human:

*“*Where there is no word and no language, there can be no dialogic relations; they cannot exist among objects or logical quantities (concepts, judgments, and so forth). *Dialogic relations* presuppose a language, but they do not reside within the system of language. They are impossible among elements of a language.” (Bakhtin 1986, p.117; emphasis added)

Bakhtin’s idea of *dialogic relations* seems to resonate with the teachings of Buber, who famously claimed that two types of such relations, *I-Thou* and *I-It*, determine who we are. It is in these two relations that we experience the world and ourselves as human beings. Our dialogue with the world reflects I-it relation whenever those we communicate with, be them other humans or inanimate objects, are treated as permanent things, already ascribed to a certain category and reducible to the sets of properties that define that category. Since in this kind of communication the object is treated as already known, as one whose place in our world has been secured once and for all, our identity is unlikely to be affected by this object in a substantial way (in this context, the word *identity* is my own, not Buber’s, but I hope it helps in conveying his idea). In contrast, entering I-Thou relation means celebrating alterity, opening oneself to the unknown Other, treating this Other as an indecomposable whole that, in the process of constant transformation, can transform us as well. It is the I-Thou relation in which our humanness finds its confirmation; it is in this relation that communication fully deserves the name “dialogue”.

After this review of philosophical approaches to dialogue (which, I admit, was way too quick and way to brief) it would be natural to say, “Paul, let’s make our picks”. Even though the set of option just presented is far from exhausting the existing possibilities, you may hesitate, feeling that each of the approaches is rich and offers something special that should not be missed. But at the closer glance, it becomes clear, at least to me, that for our present purpose, it doesn’t really matter which of the different definitions you choose. As different as the discourses of Gadamer, Bakhtin and Buber appear to be, their respective takes on the dialogue, while translated into everyday talk and deeds, seem to come down to the same thing. All of them speak about dialogue as the kind of activity in which we realize ourselves as human beings. All of them emphasize the importance of a true mutual engagement, in which one lets the Other mold one’s own self. For our communication to deserve the title of dialogue, we must make a genuine effort to see what others see, while also trying to understand why they see this the way they do.

Given this consensual portrayal, no wonder that the idea of dialogue seems to appeal to those who care about education. The natural connection between dialogue and the process of mediating children’s relations with the world has been highlighted by Buber in his essay *Education* (1925/1955). In the writings of Freire this connection never gets out of sight. Today’s educational discourse is dominated by the talk on *dialogical pedagogy* and *dialogical classroom*, with these italicized terms evocative of conversations whose participants are shying away from traditional power games and are, instead, listening carefully to each other’s arguments and giving themselves a genuine chance to get convinced by rational arguments raised by other interlocutors; or, to employ the language I am using in my research on mathematical thinking, their communication is *explorative*, as opposed to its reliance on *rituals.*

Notwithstanding the fact that in the view of its current high rating the word *dialogue* is increasingly in the danger of turning into an empty buzzword (isn’t the loss of communicative power the inescapable price of popularity?), I am prepared to declare my wholehearted commitment to the idea of dialogicality at large, and to dialogical pedagogy in particular. I guess your attitude is no different. Having said this, I am now even more concerned about the fact that our conversation lost its initial shape. I am looking at my own part in this exchange and ask myself whether it can be called “dialogical”. Was I really listening to my conversational partner (you, Paul, in case you wondered)? When voicing my disagreement, haven’t I let my ego speak lauder than my soberer self, letting it use this conversation as an opportunity to promote its own business? After all, as Buber put it, “Egos appear by setting themselves apart from other egos.” To what extent was I really open to change my positions as a result of doubts you voiced or of counterproposal you presented?

I will certainly not try to answer these questions now, in the presence of all too many pairs of eyes and ears (those of the readers who have a special penchant for discourse analysis may try to do this themselves). But I do promise to conduct a thorough soul searching while away from eavesdroppers. I wish we all do this, from time to time. After all, only too much depends on our ability to engage in a genuine dialogue, whereas the future of the dialogue depends on our ability to practice what we preach.

Well, I was unable, it seems, to end this conversation without becoming a bit of a preacher. Or simply a teacher. But, after all, this is who I really am. Fortunately, looking at all those luminaries whom I recruited in this last piece to help me in making my point, I am in a good company. And you, Paul, have been a great company all along, and you too helped me, even when disagreeing (or in fact, when opposing more than when nodding in agreement). Thank you for that, and also for the future conversation that, I hope, will follow.

Anna

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**From: Ernest, Paul**

**Tue 09/12, 13:29**

**To: Anna Sfard <sfard@netvision.net.il>**

Dear Anna

Thanks once again for your thought provoking contributions to this dialogue. Yes, it is a dialogue in my view because we are exchanging writings on a shared theme and reacting to at least some of the points of the conversational partner! Since you discuss the nature of dialogue itself I will start my response at this level too.

**What *is* dialogue?**

You pick up on the idea of dialogue itself and distinguish, I believe, between bare-bones or basic exchanges of speech/text that I shall term conversation and what might be described as ‘proper’, maybe even ethical, dialogue. Thinking about what you wrote has led me to make this distinction, which I think is a useful one. For I believe you are saying that dialogue is based on addressing the other as a fellow human being with dignity and ethical consideration (Buber, Levinas) or as someone of commensurable intelligence and epistemological power (Bakhtin, Gadamer).

This comes out when you talk about dialogical pedagogy – listening to and responding seriously to the voices of students and hence according them with human agency and dignity. Now I too favour this, but I also need to be able to encompass the broader range of discourse relations that fall under the term ‘conversation’. When the speaker is so powerful and silencing that responses are crushed, I would see this as one extreme end of conversation, a limit point if you like when one might argue that the transaction no longer retains any dialogicality or has become wholly monological.[[3]](#footnote-3) Then I would agree it is a degenerate or extreme form of conversation that loses the character of dialogue, that is of encompassing multiple voices.[[4]](#footnote-4) However, there are important examples of conversation which are not dialogical in the narrow sense given above. School examinations have a formal conversational structure. They contain the alternation of Speaker (examination questions), Answer (student response to the examination), Speaker (response to student work, grading). Thus I see this, with its formal structure as a conversation too.

We have both supervised (and examined) many master’s dissertations and doctoral theses and I regard these process as having a conversational form. In supervision, students both engage in oral exchanges with supervisors and in written exchanges (submitted drafts and supervisor suggestions) and these are of course woven together in one overall extended supervisory conversation. As supervisors we learn from these conversations, but usually not as much as the students do. What is learnt is both explicit (e.g., facts and theories) and implicit (e.g., how to write and structure academic text). These conversations are made up of a long stream of utterances – student texts responded to by supervisor texts responded to in student texts ... and so on, scores or hundreds of times. There may be dialogical sequences within this overall conversation, such as in student-supervisor dialogues during tutorial provided that there is listening and responsiveness on both sides.

There is, of course, a power differential in these conversations, as there is in many if not most conversations. After all, teaching is about conversation across power differences. We are both usually in the powerful position now, both in terms of regulating the rules of conversation, and also as epistemological authorities (not absolute but acknowledged as more knowledgeable both by our peers but more significantly by our validating institutions). Of course we (by which I mean both you and I, Anna) have served our apprenticeships as the less powerful one in many conversations. We still are in this less powerful position when we submit papers for journal publication. Just to be clear I believe that power inequalities in education and research are both potentially good (provided they are not abusive or exploitative) and are necessary to arrive at an expert consensus on what is truth or knowledge. Habermas (1987) dreamed of ideal speech situations in which power differentials between speakers do not exist or can be ignored (Day 1993, Cooren 2000). Foucault decisively critiques this on the basis that the social context with its always present power relationships can never be neutralised or ignored, it is constitutively present in all conversational situations. “[P]ower is coextensive with the social body; there are no spaces of primal liberty between the meshes of it its network” (Foucault, 1980, p. 142)

You may be growing impatient with my focus on conversation when the topic is dialogue. I am arguing that all dialogue is conversation, but not all conversational exchanges are dialogues. Let me try to at least partially define dialogue:

According to the definition I want to give, dialogue is conversation with additional characteristics.

1. Dialogue must be ethical. It necessitates that speakers address each other as fellow human beings with dignity and deserving of ethical consideration, and not as inanimate objects or lesser beings. It is the *I-Thou* relationship not *I-It* (Buber). It involves a primal responsibility for the *Other* (the partner in conversation) who is understood as a being of inexhaustible complexity, because the outcomes of their agency can never be fully predicted (Levinas). Furthermore, the Others have an ethical right to their infinite – that is inexhaustible and self-determined – agency.
2. Dialogue embodies an epistemic imperative: to listen to and respond to the utterances of partners with seriousness, that is, to engage with the content and offer reasoned responses to any claims. I would like to attribute this to Bakhtin and Gadamer but cannot find specific support for it as of now.
3. Dialogue is open, emergent and creative. Since Others have an inexhaustibly self-determined agency the outcome of a dialogue cannot be prescribed and predicted. Thus dialogue is creative, a source of new ideas and innovations.

This characterization is reflected in your talk about dialogical pedagogy – listening to and responding seriously to the voices of students and hence according them with human agency and dignity.

I said above that teaching is about dialogue across power differences. But there is an additional central feature of teaching and education, namely that from a social perspective, education is primarily about the transmission of culture.[[5]](#footnote-5) Of course there are complex issues about what elements of culture are chosen, which of these are deemed valuable for preservation and transmission. Furthermore, behind this, there is the question of who it is that decides and chooses the make-up of the curriculum. But backgrounding these contextual features, socially, education concerns the transmission of selected elements of culture. The means of doing this is conversational and incorporates two interwoven dimensions. First the presentation of cultural elements, via texts and experiences that elicit, engage and build understandings, responses, capabilities (or at least that is their intention). Secondly, there is the assessment dimension both formative and summative that feeds back evaluations of student responses so that they can shape, correct and develop their understanding and capacities to suit conventional expectations in the relevant disciplinary communities.

Both of these dimensions exemplify the power differential necessary in educational conversations. Power is manifested in the choice and mode of presentation of the content of education (including experiences and skills at all levels) as well as regulating the social theatre of transactions. Power is also enacted in the assessment of student responses and the direction of students towards the desired conventional knowledge.

You give a nice quote by Gadamer on dialogue as “a process of two people understanding each other”. Thus, it is characteristic of every dialogue that each opens himself to the other person, truly accepts his point of view as worthy of consideration and gets inside the other to such extent that he understand not a particular individual but what he says. The thing that has to be grasped is the objective rightness or otherwise of his opinion, so that they can agree with each other on the subject (*Truth & Method*, 1975; p. 347; quoted in Bernstein, 1986, p. 112)

In discussing conversation and dialogue, I would also claim that mathematical proofs are conversational. Proofs are persuasive texts that try to anticipate and accommodate possible rejoinders and critical responses by being as perfect as possible. They are submitted to referees before publication to improve them further – or rejected if irremediable weaknesses are found. Finally they are published in the broader research domain. However, they can still be challenged and revised or rejected. This process of uttering the text, listening to responses, and then improving what is shared is conversational. There are even some formal logics including Tableaux theory and Lorenzen’s proof theory that are fundamentally and explicitly conversational. (Ernest 1994).

**Accounting for imagination and non-verbal aspects of thinking**

Let me go back to the origins and main thrust of this discussion. Rereading the dialogue I noticed something I missed earlier which helps to bridge the gap between us. When you describe dialogue between different persons you include more in discourse that just the words spoken.

“My” discourse comprises all that goes into the process of communicating, and this includes activities with words, of course, but also all those unspeakables, untouchables and invisibles that we call “private/inner experience” and that give us so much conceptual headache. Sometimes, discourse (communication) may be devoid of words altogether. As I see it, when you mention conversation, you only consider the publicly accessible elements of communication.

…

But for me thinking, like communication, is not just speech, and in many cases, it is not speech at all.”

I think you caught the crux of the matter here. I was assuming that conversation and dialogue is just made up of public utterances and I recounted some of the elements of thought that are missing from this account when you converse with yourself just as a stream of words. But I did not attend fully enough to your words because I now see that you already have all the extras I was looking for in your version of discourse. I think this puts the final plank in the bridge connecting our two accounts. We are commensurable at least, if not identical, in our positions. By including all this extra (non-verbal) stuff in dialogue you provide much of what I claimed was missing from thought understood as dialogue with oneself. It provides the basis for and imaginative thinking with its imagery and potential creativity. So at this point maybe our dialogue is done – to the extent that you have answered my initial question.

However I cannot just finish here without pointing out that I believe this leaves you with a problem (a ‘conceptual headache’). Namely, how to both provide a theoretical account for, and the methodological means to study, these ‘extras’ – the non-verbal elements of discourse. In the case of embodied actions, body language, facial expressions, non-verbal noises this is not such a problem. These are dealt with in the literature. But in the case of concurrent imagery, perceptions, emotions that run alongside the verbal (and physically expressed) aspects of conversation, I think there is still and outstanding problem. However, this is not the central problem we were discussing. I guess these make up some of the cognitive (including affective) elements of what you term commognition.

So I am happy to let our dialogue finish here (with you of course having the option of a last word). For me this is a good example of what I term the third type of conversation: symbolic exchanges over extended distances and times.[[6]](#footnote-6) Traditional letter writing is an example of this form of conversation. I would also claim we have been dialoguing about dialogue! Certainly new (to me) ideas have emerged in my own writing that surprised me! And these came in response to the new ideas expressed in your contributions!

So I thank you Anna for responding to my questions and ideas, and sharing your own ideas with me too. I have gained from this dialogue both from what you have written about your ideas as well from needing to shape and develop my own thinking further in response to your challenges. There are many types of dialogue but in a dialogue that aims to be creative in the sense of extending understandings nothing is more helpful and constructive than a friendly but critical ear. You have certainly provided that for me and I hope I might possibly have offered the same to you!

Paul

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1. Ernest, P. Ed. (2018) *The Philosophy of Mathematics Education Today*, Springer. [↑](#footnote-ref-1)
2. Looking back at this it might have been useful to have another category – the Unsaid as opposed to the Unsayable. Knowledge of language will always have an unsaid dimension, but is any part of this knowledge unsayable? It certainly can’t all be said at once. [↑](#footnote-ref-2)
3. ### Interestingly, in *Women's Ways of Knowing*, Belenky et al (1986) propose *Silence* as the first of five epistemological positions through which women, it is claimed, develop as they become more epistemologically empowered. The final stage is *Constructed Knowledge: Integrating the voices* when they are able to engage in what Belenky, et al (1986, p. 144) refer to as “real talk”: the ability to listen, share and cooperate while maintaining one's own voice undiminished. Thus *silence* is seen as one of the listener positions in a conversation.

   [↑](#footnote-ref-3)
4. Encompassing multiple voices is a necessary but not sufficient condition for a dialogue. A performed written play encompasses multiple voices but is not a dialogue in my view because each response is preordained and is not a product of the agency of the answering speaker. [↑](#footnote-ref-4)
5. In addition to the social purpose of education there is the personal or individual goal of education. This is to enhance an individual’s capabilities in terms of understanding, knowledge and skills and to develop the individual as a ‘whole person’, that is, as a participant in cultures, and an independent and informed critical citizen in society. [↑](#footnote-ref-5)
6. The first type is direct face-to-face conversation in ‘real time’. The second type of conversation is intrapersonal – conversation one has with oneself in thinking. [↑](#footnote-ref-6)